- May 28, Class 10B, Week 5, Lesson A

- The following students should complete this assignment Lesson A: Kalmy and Shloimy Adka, Akilov, Beckerman, Besoussan, Pinsky, Teitelbaum, Wohlgemuth, Zagelmaum
- Read pages 103 and 104 attached
- Do problems 6,7, 9 and 10 attached
- Find attached the solution to help you with solving the problems.

Rotations

Rotations of 90° will result in a line perpendicular to the original, so the slope will be the negative reciprocal. To write the equation of a line after a 90° rotation, use the same procedure for translations and dilations, except use the negative reciprocal of the slope.

EQUATION OF THE CIRCLE

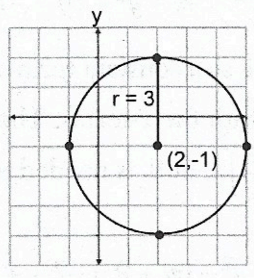
Center Radius Form of the Equation of a Circle

$(x - h)^2 + (y - k)^2 = r^2$ where the center has coordinates (h, k) and radius has length r.

- To graph a circle, first identify the center and radius from the equation. Plot a point at the center. Then plot points up, down, left, and right a distance r from the center.

Example:

Graph the equation $(x - 2)^2 + (y + 1)^2 = 9$.



The center is located at (2, –1), and $r^2 = 9$, so $r = 3$. We plot the center point at (2, –1); then plot points up, down, right, and left 3 units from the center. Use these four points as a guide to complete the circle.

General Form of the Equation of a Circle

$$x^2 + y^2 + Cx + Dy + E = 0$$

To find the coordinates of the center and the radius from the general form of the equation, you will need to convert it to the center–radius form using the following procedure:

1. Group the x-terms and y-terms on one side of the equation, and the constant on the other side of the equation.
2. Complete the square with the x-terms, and then complete the square with the y-terms.

Example:

- Find the coordinates of the center and the length of the radius of a circle whose equation is $x^2 + 4x + y^2 - 6y + 7 = 0$.

Solution:

Bring the constant term to the right.

$$x^2 + 4x + y^2 - 6y = -7$$

The coefficient of x is 4, so a constant term of $\left(\frac{4}{2}\right)^2$, or 4, is needed to complete the square with the x-terms. The coefficient of y is –6, so a constant term of $\left(\frac{-6}{2}\right)^2$, or 9, is needed to complete the square with the y-terms.

$$x^2 + 4x + 4 + y^2 - 6y + 9 = -7 + 4 + 9$$
$$(x + 2)^2 + (y - 3)^2 = 6$$

The center has coordinates (–2, 3) and the radius has a length of $\sqrt{6}$.

5. What are the coordinates of the midpoint of a segment whose endpoints have coordinates (3, 1) and (15, –7)?

(1) (27, –15) (3) (6, –4)
(2) (–6, 4) (4) (9, –3)

6. The diameter of a circle has endpoints with coordinates (4, –1) and (8, 3). Which of the following is an equation of the circle?

(1) $(x-2)^2 + (y-1)^2 = 8$ (3) $(x-6)^2 + (y-1)^2 = 8$
(2) $(x-2)^2 + (y-1)^2 = 32$ (4) $(x-6)^2 + (y-1)^2 = 32$

7. Are the segments $\overline{AB}$ and $\overline{TU}$ congruent, given coordinates $A(1, 4)$, $B(-3, 6)$, $T(2, 5)$, and $U(4, 1)$? Justify your answer.

8. Find the coordinates of the point W that divides directed segment $\overline{UV}$ in a 1:5 ratio, given coordinates $U(-3, 7)$ and $V(9, 1)$.

9. Point A has coordinates (–2, 7) and point B has coordinates (6, 3). Line m has the property that every point on the line is equidistant from points A and B. Find the equation of line m.

10. A circle is described by the equation $x^2 + 6x + y^2 - 12y + 25 = 0$. Find the radius of the circle and the coordinates of its center.

11. Circle P has a center $P(4, -5)$ and a radius with length $\sqrt{65}$. Does the point $A(8, 2)$ lie on circle P? Justify your answer.

12. Parallelogram $ABCD$ has coordinates $A(2, -1)$, $B(5, 1)$, $C(a, b)$, and $D(3, 4)$. Write the equation of the line that contains side $\overline{CD}$.

5. Apply the midpoint formula:

$$x_{MP} = \frac{1}{2}(x_1 + x_2) \qquad y_{MP} = \frac{1}{2}(y_1 + y_2)$$

$$x_{MP} = \frac{1}{2}(3 + 15) \qquad y_{MP} = \frac{1}{2}(1 + (-7))$$

$$x_{MP} = \frac{1}{2}(18) \qquad y_{MP} = \frac{1}{2}(-6)$$

$$x_{MP} = 9 \qquad y_{MP} = -3$$

The correct choice is **(4)**.

6. The center and radius of the circle are needed to write formula. The midpoint of the diameter gives the center:

$$x_{MP} = \frac{1}{2}(x_1 + x_2) \qquad y_{MP} = \frac{1}{2}(y_1 + y_2)$$

$$x_{MP} = \frac{1}{2}(4 + 8) \qquad y_{MP} = \frac{1}{2}(-1 + 3)$$

$$x_{MP} = \frac{1}{2}(12) \qquad y_{MP} = \frac{1}{2}(2)$$

$$x_{MP} = 6 \qquad y_{MP} = 1$$

The radius is the distance from the center point to eithe endpoint of the diameter. Apply the distance formula with point (6, 1) and (8, 3).

$$\begin{aligned} \text{distance} &= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \\ &= \sqrt{(8-6)^2 + (3-1)^2} \\ &= \sqrt{(2)^2 + (2)^2} \\ &= \sqrt{8} \end{aligned}$$

The radius of the circle is $\sqrt{8}$, and its center has coordinates (6, 1). Substitute these values for r, h, and k in the equation of a circle:

$$(x - h)^2 + (y - k)^2 = R^2$$
$$(x - 6)^2 + (y - 1)^2 = \sqrt{8}^2$$
$$(x - 6)^2 + (y - 1)^2 = 8$$

The correct choice is **(3)**.

7. Two segments are congruent if their lengths are equal, so apply the distance formula to determine the length of each segment.

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$AB = \sqrt{(-3-1)^2 + (6-4)^2} \qquad TU = \sqrt{(4-2)^2 + (1-5)^2}$$

$$= \sqrt{(-4)^2 + (2)^2} \qquad = \sqrt{(2)^2 + (-4)^2}$$

$$= \sqrt{16+4} \qquad = \sqrt{4+16}$$

$$= \sqrt{20} \qquad = \sqrt{20}$$

$AB = TU$; therefore, the 2 segments are congruent.

8. $$\frac{UW}{WV} = \frac{1}{5} = \frac{x-(-3)}{9-x}$$

$$9 - x = 5(x + 3)$$

$$9 - x = 5x + 15$$

$$-6 = 6x$$

$$x = -1$$

Repeating for the y-coordinate:

$$\frac{UW}{WV} = \frac{1}{5} = \frac{y-7}{1-y}$$

$$1 - y = 5(y - 7)$$

The coordinates of W are $(-1, 6)$.

9. Line m is the line of reflection that maps A to B, so it must be the perpendicular bisector of $\overline{AB}$. To find the perpendicular bisector, calculate the midpoint and slope of $\overline{AB}$. Then write the equation of the line with the negative reciprocal slope that passes through the midpoint.

$$x_{MP} = \frac{1}{2}(x_1 + x_2),\ y_{MP} = \frac{1}{2}(y_1 + y_2)$$

$$x_{MP} = \frac{1}{2}(-2 + 6),\ y_{MP} = \frac{1}{2}(7 + 3)$$

$$x_{MP} = 2, \qquad y_{MP} = 5$$

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\text{slope}_{AB} = \frac{3-7}{6-(-2)}$$

$$= -\frac{1}{2}$$

$\text{slope}_{\text{line } m} = 2$ — ⊥ lines have negative reciprocal slopes

$y - y_1 = m(x - x_1)$ — point-slope equation of a line

$y - 5 = 2(x - 2)$ or $y = 2x + 1$ — substitute the coordinates of the midpoint for x_1 and y_1, and 2 for m

10. Apply the completing the square procedure to rewrite the circle in $(x - h)^2 + (y - k)^2 = R^2$ form. Rewrite the equation with the variables on the left and constant on the right.

$$x^2 + 6x + y^2 - 12y + 25 = 0$$
$$x^2 + 6x + y^2 - 12y = -25$$

The constant needed to complete the square is $\left(\frac{1}{2}b\right)^2$, where b is the coefficient of the linear x- and y-terms. For the x-terms, the necessary constant is $\left(\frac{1}{2}(6)\right)^2$, or 9. For the y-terms $\left(\frac{1}{2}(-12)\right)^2$, or 36, is needed. Add the required constants to each side of the equation.

$$x^2 + 6x + 9 + y^2 - 12y + 36 = -25 + 9 + 36$$

$(x + 3)^2 + (y - 6)^2 = 20$ — factor the x-terms and the y-terms

The center has coordinates $(-3, 6)$ and the radius is $\sqrt{20}$.