June 4 class 11B Lesson A: Beckerman, Elharar, Klein, Mimni, Soleynabzadeh, Sussman, Tendler.

Assignment: Read prisms, cylinders, cones, pyramids and spheres: pages 213-215

Turn in problems 1-5: pages 225 and 226. Pages 231 and 232 have the solutions to help you solve the problems, if you need help.

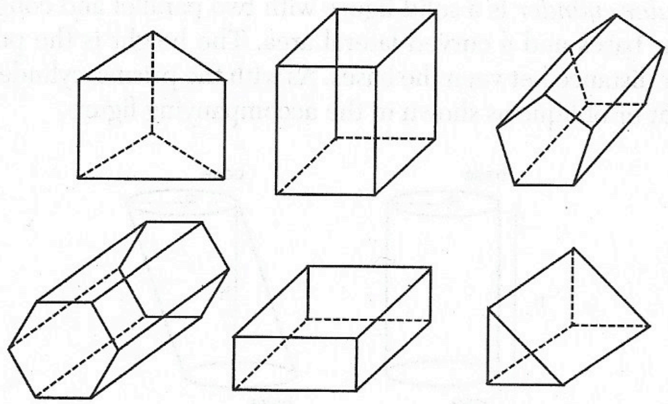
3.12 SOLIDS

DEFINITIONS

- A *solid* is any 3-D figure that is fully enclosed.
- A *face* of a solid is any of the surfaces that bound the solid.
- A *polyhedron* is any solid whose faces are polygons.
- An *edge* is the intersection of two faces in a polyhedron.

PRISMS

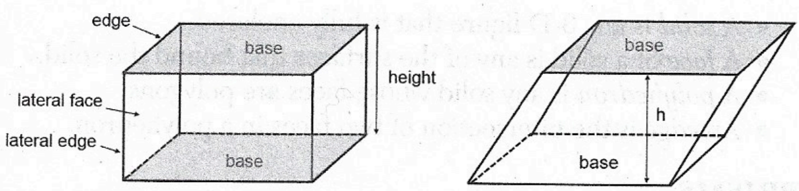
A *prism* is a polyhedron with two congruent, parallel polygons for bases. The bases of a prism can have any shape. The accompanying figure shows 6 different prisms.



When working with prisms, keep in mind the following facts:

- The height of a prism, h, is the distance between the two bases shown in the accompanying figure.
- The lateral faces are all the faces other than the two parallel bases.
- A right prism has lateral edges that are perpendicular to the bases and lateral faces that are rectangles.
- The lateral edges of an oblique prism are not perpendicular to the bases, and the lateral faces are parallelograms.

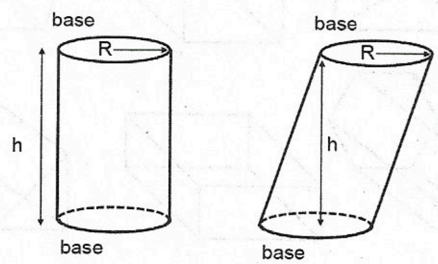
- The volume of any prism is found with the formula $V = Bh$, where B is the area of the base.



right prism oblique prism

CYLINDERS

A *circular cylinder* is a solid figure with two parallel and congruent circular bases and a curved lateral area. The height is the perpendicular distance between the bases. As with the prisms, cylinders can be right or oblique as shown in the accompanying figure.

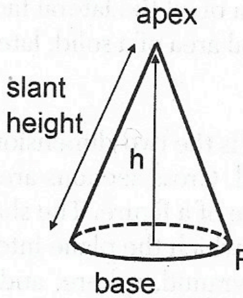


right circular cylinder oblique circular cylinder

The volume of a cylinder is given by $V = Bh$, where B is the area of the base. In a circular cylinder, the circular base has an area of πR^2, so the volume formula can be rewritten as $V = \pi R^2 h$.

CONES AND PYRAMIDS

A *circular cone* is a solid with one circular base that comes to a point at an apex. A *pyramid* is a polyhedron having one polygonal base and triangles for lateral faces. The base of a pyramid can be any polygon, and the lateral faces are all triangles. The height of cones and pyramids is the perpendicular distance from the apex to the base. The slant height is the distance along the lateral surface perpendicular to the perimeter of the base.

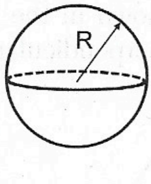


The volume of pyramids and cones are found from the same formula:

$V = \frac{1}{3}Bh$ where B is the area of the base.

SPHERES

A *sphere* is the set of points a fixed distance from a center point, and its volume is given by the formula $V = \frac{4}{3}\pi R^3$.



SURFACE AREA AND LATERAL AREA

The *surface area* is the area of all faces of a solid. Surface area can be found by calculating the area of all the faces of a solid individually and then summing them. You should be able to calculate the surface area of cubes, prisms, and pyramids since the faces of these solids are all polygons. Surface area of cones, cylinders, and spheres involve curved surfaces and are outside the scope of this course.

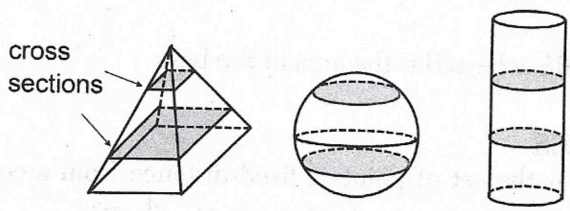
The lateral area of a solid excludes the bases. For a prism, do not include the two parallel bases. For a pyramid, exclude the one base.

Lateral face—any face of a solid other than its bases
Lateral area—the area of all the lateral faces of a solid
Surface area—the total area of a solid, lateral area + area of bases

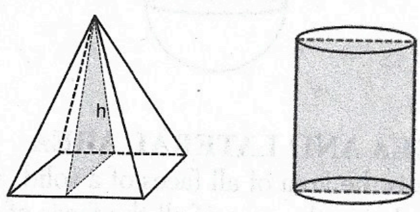
CROSS-SECTIONS

A cross-section of a solid is the two-dimensional figure created when a plane intercepts a solid. Cross-sections are often taken parallel or perpendicular to the base of a figure. The shape of the cross-section depends on the angle at which the plane intersects the solid.

Cross-sections of a pyramid, sphere, and cylinder are shown in the accompanying figure.



A cross-section of a pyramid taken perpendicular to the base will be shaped triangular, as shown in the figure, or trapezoidal. The cross-section of a cylinder perpendicular to the base is a rectangle.



Practice Exercises

1. A basketball has a diameter of 9.6 inches. What is its volume?

(1) 72.3 in^3
(3) 463 in^3
(2) 347 in^3
(4) 3706 in^3

2. A right circular cylinder has a volume of 200 cm^3. If the height of the cylinder is 6 cm, what is the radius of the cylinder? Round your answer to the nearest hundredth.

(1) 2.9 cm
(3) 3.3 cm
(2) 3.0 cm
(4) 3.8 cm

3. Jenna is preparing to plant a new lawn and has a pile of topsoil delivered to her house. A dump truck delivers the topsoil and dumps it in a pile that is cone shaped. The radius of the pile is 9 ft, and the height is 3 ft. If the topsoil has a density of 95 lb/ft^3, what is the weight of the topsoil?

(1) 18,764 lb
(3) 28,546 lb
(2) 24,175 lb
(4) 32,676 lb

4. A right circular cylinder has a radius of 4 and a height of 10. What is the area of a cross-section taken parallel to, and halfway between, the bases? Write your answer in terms of π.

(1) 4π
(3) 12π
(2) 8π
(4) 16π

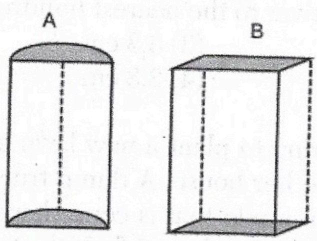
5. A cylinder and a cone have the same volume. If each solid has the same height, what is the ratio of the radius of the cylinder to the radius of the cone?

(1) 2:1
(2) 3:1
(3) $\sqrt{2}$:2
(4) $\sqrt{3}$:3

6. $\triangle JKL$ is a right triangle with a right angle at K, $JK = 6$, and $KL = 10$. Which of the following solids is generated with $\triangle JKL$ rotated 360° about KL?

(1) a cylinder with a height of 10 and a diameter of 12
(2) a cylinder with a height of 6 and a diameter of 20
(3) a right circular cone with a height of 10 and a diameter of 12
(4) a right circular cone with a height of 6 and a diameter of 20

7. Solids A and B, shown in the figure below, each have uniform cross-sections perpendicular to the shaded bases.



If the heights of the two solids are equal, which of the followin[g] statements represents an application of Cavalieri's principle?

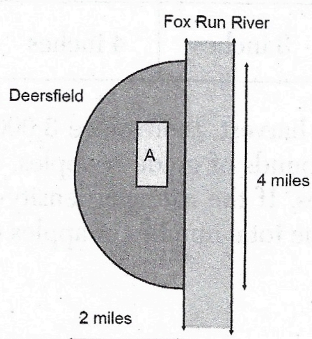
(1) The surface areas of solid A and solid B may be different even though their volumes are equal.
(2) Solid A can be generated by rotating a rectangle, where[as] solid B is not a solid of revolution.
(3) If the area of base A equals the area of base B, then th[e] volumes of the solids are equal.
(4) If the volume and height of solid A and B are equal, the[n] their surface areas must be equal.

8. The base of a cone is described by the curve $x^2 + y^2 = 16$. If the altitude of the cone intersects the center of its base, and the volume of the cone is 72π, what is the height of the cone?

9. A cylindrical piece of metal has a radius of 15 in and a height of 2 in. It goes through a hot-press machine that reduces the height of the cylinder to $\frac{1}{4}$ in. What is the new radius of the cylinder, assuming no material is lost?

10. Jack is making a scaled drawing of the floor plan of his home. The scale factor is 1 in:4 ft. The drawing of his living room is a rectangle measuring 5 inches by 3 inches. He is planning to purchase new carpet for the living room that costs \$4 per square foot. How much will the carpet cost?
(1) \$720
(2) \$840
(3) \$960
(4) \$1,040

11. The city of Deersfield sits along the bank of the Fox Run River. A map of the city is shown below. The downtown region, indicated by region A, is $\frac{1}{2}$ mile wide and 1 mile long. The population density of Deersfield, except for the downtown region, is 800 people per square mile. The population density of the downtown region is 4,000 people per square mile. What is the population of Deersfield? Round to the nearest whole number.



Solutions

1. The basketball can be modeled as a sphere whose volume is $\frac{4}{3}\pi R^3$. The radius equals half the diameter, or 4.8 in.

$$\begin{aligned} V &= \frac{4}{3}\pi R^3 \\ &= \frac{4}{3}\pi(4.8 \text{ in})^3 \\ &= 463 \text{ in}^3 \end{aligned}$$

The correct choice is **(3)**.

2. Start with the formula for volume, substitute the known values, and then solve for the radius.

$$\begin{aligned} V &= B \cdot h \\ V &= \pi R^2 h \\ 200 &= \pi(R^2)(6) \\ R^2 &= 10.61032 \\ R &= \sqrt{10.61032} \\ &= 3.3 \text{ cm} \end{aligned}$$

The correct choice is **(3)**.

3. To find the weight of the topsoil, we need to first find the volume of the pile. The volume of a cone is $\frac{1}{3}Bh$, where B is the area of the base. This requires finding the area of the circular base.

$$\begin{aligned} B &= \pi R^2 \\ &= \pi(9^2) \\ &= 254.4690 \text{ ft}^2 \end{aligned}$$

We are now ready to find the volume.

$$\begin{aligned} V &= \frac{1}{3}B \cdot h \\ &= \frac{1}{3}(254.4690)(3) \\ &= 254.469 \text{ ft}^3 \end{aligned}$$

Now calculate the weight using the volume and density.

$$\begin{aligned}\text{weight} &= V \cdot \text{density}\\ &= 254.469\ \text{ft}^3 \cdot 95\ \text{lb/ft}^3\\ &= 24{,}174.55\ \text{lb}\\ &= 24{,}175\ \text{lb}\end{aligned}$$

The correct choice is **(2)**.

4. The cross-section will be congruent to the base, so it is a circle with radius of 4. The area is πR^2, or 16π.

The correct choice is **(4)**.

5. The strategy is to set the two volume formulas equal to each other and rearrange, solving for the ratio of the radii.

$$V_{\text{cone}} = \frac{1}{3}\pi R_{\text{cone}}^2$$

$$V_{\text{cylinder}} = \pi R_{\text{cylinder}}^2 h$$

$$V_{\text{cylinder}} = V_{\text{cone}}$$

$$\pi R_{\text{cylinder}}^2 h = \frac{1}{3}\pi R_{\text{cone}}^2 h$$

Now solve for the ratio $\dfrac{R_{\text{cylinder}}}{R_{\text{cone}}}$

$$R_{\text{cylinder}}^2 = \frac{1}{3}R_{\text{cone}}^2 \qquad \text{divide by } \pi h$$

$$\frac{R_{\text{cylinder}}^2}{R_{\text{cone}}^2} = \frac{1}{3} \qquad \text{divide by } R_{\text{cone}}^2$$

$$\frac{R_{\text{cylinder}}}{R_{\text{cone}}} = \frac{1}{\sqrt{3}} \qquad \text{take square root of each side}$$

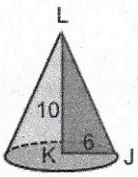
The ratio does not match any of the choices, so try rationalizing the denominator. To do so, multiply the numerator and denominator by $\sqrt{3}$.

$$\frac{R_{\text{cylinder}}}{R_{\text{cone}}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$\frac{R_{\text{cylinder}}}{R_{\text{cone}}} = \frac{\sqrt{3}}{3}$$

The correct choice is **(4)**.

6. A right triangle rotated 360° about one of its legs will generate a cone.



The leg aligned with the axis of rotation becomes the height, so the height equals 10. The leg perpendicular to the axis of rotation becomes the radius, so the radius equals 6 and the diameter equals 12.

The correct choice is **(3)**.

7. Cavalieri's principle states that two parallel planes will intercept the same volume in two solids if the cross-sectional areas are uniform and equal. Choice 3 represents this principle.

The correct choice is **(3)**.

8. $x^2 + y^2 = 16$ describes a circle centered at the origin with a radius of 4. Its area is

$$A = \pi R^2 = 16\pi$$

Apply the volume formula of the cone to find the height.

$$V = \frac{1}{3}Bh$$

$$72\pi = \frac{1}{3}16\pi h$$

$$h = 72 \cdot 3 \cdot \frac{1}{16}$$

$$= 13.5$$

9. Find the volume of the cylinder before pressing and set it equal to the expression for the volume after pressing. Use this equation to solve for the radius after pressing.

$$V = \pi R^2 h$$

$$V_{\text{before}} = \pi \cdot 15^2 \cdot 2$$

$$= 450\pi \text{ in}^3$$

$$V_{\text{after}} = \pi \cdot R^2 \cdot \frac{1}{4} \qquad \text{we don't know the new radius}$$

$$V_{\text{before}} = V_{\text{after}}$$

$$\pi \cdot R^2 \cdot \frac{1}{4} = 450\pi \qquad \text{volumes must be equal}$$

$$\frac{R^2}{4} = 450$$

$$R^2 = 1{,}800$$

$$R = \sqrt{1{,}800}$$

$$= 42.426 \text{ in}$$