

- May 28, Class 11B, Week 5, Lesson A
- The following students should complete this assignment Lesson A:Beckerman, Elharar, Klein, Mimni, Soleynabzadeh, Sussman, Tendler read page 103 and 104 attached
- Do problems 6,7, 9 and 10 attached
- Find attached the solutions to help you with solving the problems.

**Rotations**

Rotations of  $90^\circ$  will result in a line perpendicular to the original, so the slope will be the negative reciprocal. To write the equation of a line after a  $90^\circ$  rotation, use the same procedure for translations and dilations, except use the negative reciprocal of the slope.

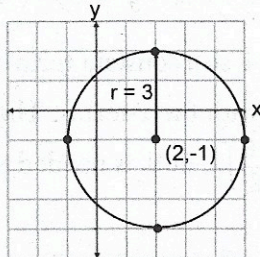
**EQUATION OF THE CIRCLE****Center Radius Form of the Equation of a Circle**

$(x - h)^2 + (y - k)^2 = r^2$  where the center has coordinates  $(h, k)$  and radius has length  $r$ .

- To graph a circle, first identify the center and radius from the equation. Plot a point at the center. Then plot points up, down, left, and right a distance  $r$  from the center.

**Example:**

Graph the equation  $(x - 2)^2 + (y + 1)^2 = 9$ .



The center is located at  $(2, -1)$ , and  $r^2 = 9$ , so  $r = 3$ . We plot the center point at  $(2, -1)$ ; then plot points up, down, right, and left 3 units from the center. Use these four points as a guide to complete the circle.

**General Form of the Equation of a Circle**

$$x^2 + y^2 + Cx + Dy + E = 0$$

To find the coordinates of the center and the radius from the general form of the equation, you will need to convert it to the center-radius form using the following procedure:

1. Group the  $x$ -terms and  $y$ -terms on one side of the equation, and the constant on the other side of the equation.
2. Complete the square with the  $x$ -terms, and then complete the square with the  $y$ -terms.

**Example:**

- Find the coordinates of the center and the length of the radius of a circle whose equation is  $x^2 + 4x + y^2 - 6y + 7 = 0$ .

*Solution:*

Bring the constant term to the right.

$$x^2 + 4x + y^2 - 6y = -7$$

The coefficient of  $x$  is 4, so a constant term of  $\left(\frac{4}{2}\right)^2$ , or 4, is needed to complete the square with the  $x$ -terms. The coefficient of  $y$  is  $-6$ , so a constant term of  $\left(\frac{-6}{2}\right)^2$ , or 9, is needed to complete the square with the  $y$ -terms.

$$x^2 + 4x + 4 + y^2 - 6y + 9 = -7 + 4 + 9$$

$$(x + 2)^2 + (y - 3)^2 = 6$$

The center has coordinates  $(-2, 3)$  and the radius has a length of  $\sqrt{6}$ .

**106      A Brief Review of Key Geometry Facts and Skills**

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5. What are the coordinates of the midpoint of a segment whose endpoints have coordinates  $(3, 1)$  and  $(15, -7)$ ?
- (1)  $(27, -15)$                       (3)  $(6, -4)$   
(2)  $(-6, 4)$                           (4)  $(9, -3)$
6. The diameter of a circle has endpoints with coordinates  $(4, -1)$  and  $(8, 3)$ . Which of the following is an equation of the circle?
- (1)  $(x - 2)^2 + (y - 1)^2 = 8$       (3)  $(x - 6)^2 + (y - 1)^2 = 8$   
(2)  $(x - 2)^2 + (y - 1)^2 = 32$       (4)  $(x - 6)^2 + (y - 1)^2 = 32$
7. Are the segments  $\overline{AB}$  and  $\overline{TU}$  congruent, given coordinates  $A(1, 4)$ ,  $B(-3, 6)$ ,  $T(2, 5)$ , and  $U(4, 1)$ ? Justify your answer.
8. Find the coordinates of the point  $W$  that divides directed segment  $\overline{UV}$  in a 1:5 ratio, given coordinates  $U(-3, 7)$  and  $V(9, 1)$ .
9. Point  $A$  has coordinates  $(-2, 7)$  and point  $B$  has coordinates  $(6, 3)$ . Line  $m$  has the property that every point on the line is equidistant from points  $A$  and  $B$ . Find the equation of line  $m$ .
10. A circle is described by the equation  $x^2 + 6x + y^2 - 12y + 25 = 0$ . Find the radius of the circle and the coordinates of its center.
11. Circle  $P$  has a center  $P(4, -5)$  and a radius with length  $\sqrt{65}$ . Does the point  $A(8, 2)$  lie on circle  $P$ ? Justify your answer.
12. Parallelogram  $ABCD$  has coordinates  $A(2, -1)$ ,  $B(5, 1)$ ,  $C(a, b)$ , and  $D(3, 4)$ . Write the equation of the line that contains side  $\overline{CD}$ .

5. Apply the midpoint formula:

$$x_{\text{MP}} = \frac{1}{2}(x_1 + x_2) \quad y_{\text{MP}} = \frac{1}{2}(y_1 + y_2)$$

$$x_{\text{MP}} = \frac{1}{2}(3 + 15) \quad y_{\text{MP}} = \frac{1}{2}(1 + (-7))$$

$$x_{\text{MP}} = \frac{1}{2}(18) \quad y_{\text{MP}} = \frac{1}{2}(-6)$$

$$x_{\text{MP}} = 9 \quad y_{\text{MP}} = -3$$

The correct choice is (4).

6. The center and radius of the circle are needed to write formula. The midpoint of the diameter gives the center:

$$x_{\text{MP}} = \frac{1}{2}(x_1 + x_2) \quad y_{\text{MP}} = \frac{1}{2}(y_1 + y_2)$$

$$x_{\text{MP}} = \frac{1}{2}(4 + 8) \quad y_{\text{MP}} = \frac{1}{2}(-1 + 3)$$

$$x_{\text{MP}} = \frac{1}{2}(12) \quad y_{\text{MP}} = \frac{1}{2}(2)$$

$$x_{\text{MP}} = 6 \quad y_{\text{MP}} = 1$$

The radius is the distance from the center point to either endpoint of the diameter. Apply the distance formula with point (6, 1) and (8, 3).

$$\begin{aligned} \text{distance} &= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \\ &= \sqrt{(8 - 6)^2 + (3 - 1)^2} \\ &= \sqrt{(2)^2 + (2)^2} \\ &= \sqrt{8} \end{aligned}$$

The radius of the circle is  $\sqrt{8}$ , and its center has coordinates (6, 1). Substitute these values for  $r$ ,  $h$ , and  $k$  in the equation of a circle:

$$(x - h)^2 + (y - k)^2 = R^2$$

$$(x - 6)^2 + (y - 1)^2 = \sqrt{8}^2$$

$$(x - 6)^2 + (y - 1)^2 = 8$$

The correct choice is (3).

7. Two segments are congruent if their lengths are equal, so apply the distance formula to determine the length of each segment.

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$\begin{aligned} AB &= \sqrt{(-3-1)^2 + (6-4)^2} & TU &= \sqrt{(4-2)^2 + (1-5)^2} \\ &= \sqrt{(-4)^2 + (2)^2} & &= \sqrt{(2)^2 + (-4)^2} \\ &= \sqrt{16+4} & &= \sqrt{4+16} \\ &= \sqrt{20} & &= \sqrt{20} \end{aligned}$$

$AB = TU$ ; therefore, the 2 segments are congruent.

$$\begin{aligned} 8. \quad \frac{UW}{WV} &= \frac{1}{5} = \frac{x - (-3)}{9 - x} \\ 9 - x &= 5(x + 3) \\ 9 - x &= 5x + 15 \\ -6 &= 6x \\ x &= -1 \end{aligned}$$

Repeating for the  $y$ -coordinate:

$$\begin{aligned} \frac{UW}{WV} &= \frac{1}{5} = \frac{y - 7}{1 - y} \\ 1 - y &= 5(y - 7) \end{aligned}$$

The coordinates of  $W$  are  $(-1, 6)$ .

9. Line  $m$  is the line of reflection that maps  $A$  to  $B$ , so it must be the perpendicular bisector of  $\overline{AB}$ . To find the perpendicular bisector, calculate the midpoint and slope of  $\overline{AB}$ . Then write the equation of the line with the negative reciprocal slope that passes through the midpoint.

$$\begin{aligned} x_{MP} &= \frac{1}{2}(x_1 + x_2), \quad y_{MP} = \frac{1}{2}(y_1 + y_2) \\ x_{MP} &= \frac{1}{2}(-2 + 6), \quad y_{MP} = \frac{1}{2}(7 + 3) \\ x_{MP} &= 2, \quad y_{MP} = 5 \end{aligned}$$

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\begin{aligned}\text{slope}_{AB} &= \frac{3-7}{6-(-2)} \\ &= -\frac{1}{2}\end{aligned}$$

$$\text{slope}_{\text{line } m} = 2$$

⊥ lines have negative reciprocal slopes

$$y - y_1 = m(x - x_1)$$

point-slope equation of a line

$$y - 5 = 2(x - 2) \text{ or } y = 2x + 1$$

substitute the coordinates of the midpoint for  $x_1$  and  $y_1$ , and 2 for  $m$

**10.** Apply the completing the square procedure to rewrite the circle in  $(x - h)^2 + (y - k)^2 = R^2$  form. Rewrite the equation with the variables on the left and constant on the right.

$$\begin{aligned}x^2 + 6x + y^2 - 12y + 25 &= 0 \\ x^2 + 6x + y^2 - 12y &= -25\end{aligned}$$

The constant needed to complete the square is  $\left(\frac{1}{2}b\right)^2$ , where  $b$  is the coefficient of the linear  $x$ - and  $y$ -terms. For the  $x$ -terms, the necessary constant is  $\left(\frac{1}{2}(6)\right)^2$ , or 9. For the  $y$ -terms  $\left(\frac{1}{2}(-12)\right)^2$ , or 36, is needed. Add the required constants to each side of the equation.

$$x^2 + 6x + 9 + y^2 - 12y + 36 = -25 + 9 + 36$$

$$(x + 3)^2 + (y - 6)^2 = 20 \quad \text{factor the } x\text{-terms and the } y\text{-terms}$$

The center has coordinates  $(-3, 6)$  and the radius is  $\sqrt{20}$ .