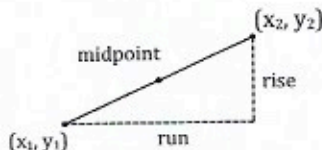## 3.6 COORDINATE GEOMETRY

COORDINATE GEOMETRY FORMULAS

Given two points (x_1, y_1) and (x_2, y_2):

- The distance between the points is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

- The midpoint of the segment joining the points is
$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

- The slope of the segment joining the points is $\dfrac{y_2 - y_1}{x_2 - x_1}$,

    or $\dfrac{rise}{run}$

DIVIDING A SEGMENT PROPORTIONALLY

A directed segment is one that has a specified starting point and ending point. We can divide a directed segment into parts that are in any given ratio using the two proportions ratio $= \dfrac{x - x_1}{x_2 - x}$ and ratio $= \dfrac{y - y_1}{y_2 - y}$.

For example, given $J(1, -2)$ and $K(11, 3)$, find the coordinate (x, y) of point L that divides $\overline{JK}$ in a 2:3 ratio.
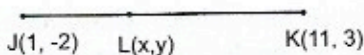
- Use the coordinates of J for point 1 and the coordinates of K for point 2.

$$x_1 = 1 \qquad x_2 = 11 \qquad y_1 = -2 \text{ and } y_2 = 3$$

- Apply the formula for the x-coordinate.

$$\text{ratio} = \frac{x - x_1}{x_2 - x}$$

$$\frac{2}{3} = \frac{x-1}{11-x}$$
$$2(11 - x) = 3(x - 1)$$
$$22 - 2x = 3x - 3$$
$$25 = 5x$$
$$x = 5$$

- Repeat the process for the y-coordinate.

$$\text{ratio} = \frac{y - y_1}{y_2 - y}$$

$$\frac{2}{3} = \frac{y - (-2)}{3 - y}$$
$$3y + 6 = 6 - 2y$$
$$5y = 0$$
$$y = 0$$

Point $L(5, 0)$ divides $\overline{JK}$ in a 2:3 ratio.

AREA AND PERIMETER

The lengths found with the distance formula can be used to calculate the perimeter and area of figures. If the figure is irregular, three strategies can be used:

- Divide the figure into shapes whose areas can be calculated easily (squares, rectangles, triangles, trapezoids, and circles).
- Sketch a bounding rectangle around the figure. Calculate the area of the rectangle, and then subtract the area of all the triangles that fall outside the figure but with the rectangle.
- If the figure has curves, estimate the area by modeling the curved sides with straight segments. The more segments that are used to model a curve, the more accurate the result will be.